A POLYHEDRAL FINITE ELEMENT FORMULATION USING PROJECTED GRADIENTS AND THE DUAL BASIS WITH APPLICATIONS TO NONLINEAR SOLID MECHANICS

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Several types of shape functions are now available for use in defining $H^1$-conforming polyhedral finite elements. These shape functions are more generally known as generalized barycentric coordinates (GBC) since they form a partition of unity and can reproduce linear functions. Even with the availability of shape functions for general polyhedra, one key challenge remaining for a viable polyhedral finite element formulation for nonlinear solid mechanics is developing an efficient and consistent quadrature scheme that results in a stable discretization. Quadrature for polyhedral elements with GBC shape functions is especially challenging given their non-polynomial nature. Development of an efficient quadrature scheme is especially crucial for applications in nonlinear solid mechanics where material constitutive models can be extremely expensive, e.g. for materials with a large number of internal state variables or for plasticity models with complex yield surfaces.

In this work, a derivative correction scheme is proposed that ensures the necessary integration consistency in order to pass the engineering patch test. The correction scheme involves projecting the true shape-function derivatives to a finite-dimensional subspace. This projection results in the exact satisfaction of the patch test. The basis and dimension of the correction subspace determines the stability of the overall discretization. The derivative projection is facilitated through the use of the conjugate (dual) basis and covariant coordinates. There are several possibilities in choosing the correction subspace including using the given shape-function basis directly. Several verification and nonlinear demonstration examples are presented along with observed rates of convergence.

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