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IMPROVED MULTISCALE FINITE ELEMENT METHODS FOR ADVECTION-DIFFUSION PROBLEMS

Rutger A. Biezemans

CERMICS and MATHERIALS project-team, École des Ponts and Inria
6 et 8 avenue Blaise Pascal, 77455 Marne-La-Vallée cedex 2, France
rutger.biezemans@enpc.fr, https://cermics-lab.enpc.fr/rutger-biezemans/

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The multi-scale finite element method (MsFEM) is a finite element (FE) approach that allows to solve partial differential equations (PDEs) with highly oscillatory coefficients on a coarse mesh, i.e. a mesh with elements of size much larger than the characteristic scale of the heterogeneities [1]. To do so, MsFEMs use pre-computed basis functions, adapted to the differential operator, and that take into account the small scales of the problem.

When the PDE contains dominating advection terms, naive finite element approximations lead to spurious oscillations, even in the absence of oscillatory coefficients. Stabilization techniques are to be adopted, see e.g. [4, Chapter 13]. In spite of various proposals to combine multi-scale and stabilization methods [3], a universally best method has not yet been identified.

The main contribution of this talk will be the addition of suitable bubble functions to the approximation space of previously studied MsFEMs. The extent to which an MsFEM is stabilized by these bubbles turns out to vary greatly from one MsFEM variant to another. In particular, we will build on an adaptation of the Crouzeix-Raviart finite element in the MsFEM-spirit [2], for which the stabilization by bubble functions is much more effective than for other MsFEM variants. Differences in the performance of various methods will be illustrated by numerical experiments.

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References

[1] T. Y. Hou and X.-H. Wu. A multiscale finite element method for elliptic problems in composite materials and porous media. *Journal of Computational Physics*, 134(1):169–189, 1997.

[2] C. Le Bris, F. Legoll, and A. Lozinski. MsFEM à la Crouzeix-Raviart for Highly Oscillatory Elliptic Problems. *Chinese Annals of Mathematics, Series B*, 34(1):113–138, 2013.

[3] C. Le Bris, F. Legoll, and F. Madiot. A numerical comparison of some multiscale finite element approaches for advection-dominated problems in heterogeneous media. *ESAIM: M2AN*, 51(3):851–888, 2017.

[4] A. Quarteroni. *Numerical Models for Differential Problems*, volume 16 of *MS&A*. Springer International Publishing, Cham, 3rd edition, 2017.